Symbolic Model Checking for Dynamic Epistemic Logic: Factual Change and Bisimulations

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From Computation to Agency
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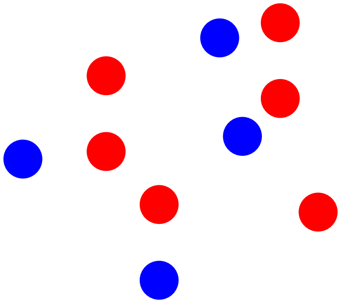
Abstract

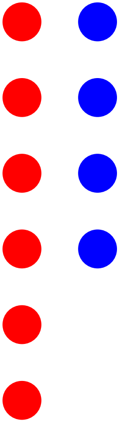
Dynamic Epistemic Logic (DEL) is a widely studied framework to reason about knowledge and dynamics. In particular it offers an intuitive way to describe information change in multi-agent settings. Its practical applications however are limited so far, possibly because there are no efficient implementations of standard decision problems like model checking and satisfiability. This is in contrast to temporal logics and interpreted systems where both theoretical work on symbolic representation and practical tools abound.

Recently we developed symbolic model checking methods for DEL, starting with simple S5 public announcement logic and by now also covering beliefs and action models.

In this talk I will recap the general ideas and present two new parts of this research: What bisimulations look like between symbolic structures and how we can model factual change.



Are there more red or more blue points?



Are there more red or more blue points?

$6\times$  $4\times$ 

Are there more red or more blue points?

Representation matters!

1. **Dynamic Epistemic Logic**
2. Symbolic Model Checking
3. Factual Change
4. Symbolic Bisimulations

Epistemic Logic

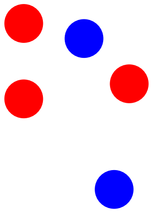
Syntax

$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi$

Kripke Models

$\mathcal{M} = (W, R_i, \mathrm{Val})$ where

- W set of worlds
- $R_i \subseteq W \times W$ indistinguishability relation
- $\mathrm{Val}\colon W \to \mathcal{P}(P)$ valuation function



Semantics

$\mathcal{M}, w \models K_i\varphi$ iff wR_iv implies $\mathcal{M}, v \models \varphi$

Dynamic Epistemic Logic: Action Models

Action Models

$\mathcal{A} = (A, R, \text{pre}, \text{post})$ where

- A set of atomic events
- $R_i \subseteq A \times A$ indistinguishability relation
- $\text{pre}\colon A \to \mathcal{L}$ precondition function
- $\text{post}\colon A \to P \to \mathcal{L}$ postcondition function

Product Update

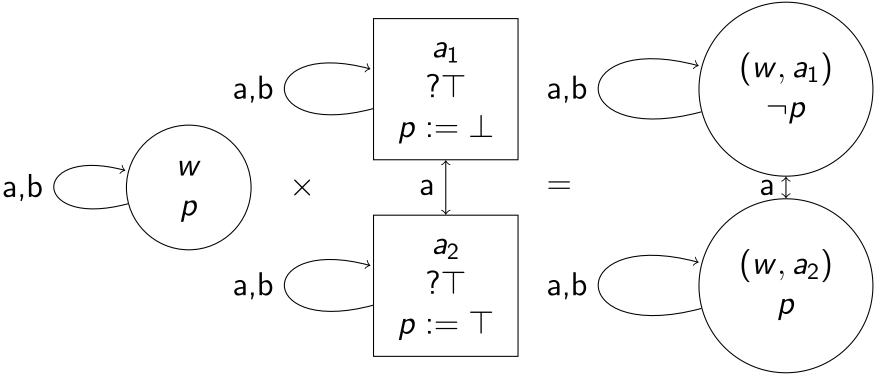
$\mathcal{M} \times \mathcal{A} := (W^{\text{new}}, \mathcal{R}_i^{\text{new}}, \text{Val}^{\text{new}})$ where

- $W^{\text{new}} := \{(w, a) \in W \times A \mid \mathcal{M}, w \vDash \text{pre}(a)\}$
- $\mathcal{R}_i^{\text{new}} := \{((w, a), (v, b)) \mid \mathcal{R}_i wv \text{ and } R_i ab\}$
- $\text{Val}^{\text{new}}((w, a)) := \{p \in V \mid \mathcal{M}, w \vDash \text{post}_a(p)\}$

$\mathcal{M}, v \vDash [\mathcal{A}, a]\varphi$ iff $\mathcal{M}, w \vDash \text{pre}(a)$ implies $(\mathcal{M} \times \mathcal{A}, (w, a)) \vDash \varphi$

(Baltag, Moss, and Solecki 1998), (van Benthem, van Eijck, and Kooi 2006)

DEL Example: Coin Flip hidden from a



$$\mathcal{M}, w \models K_a p \wedge K_b p \wedge [\mathcal{A}, a_1](K_b \neg p \wedge \neg K_a \neg p)$$

DEL compared to ATL, ETL, CTL, ...

- DEL: events are *model changing* operations
- TL: time is a *relation inside the model*

For a detailed comparison, see (van Benthem et al. 2009) and (van Ditmarsch, Hoek, and Ruan 2013).

1. Dynamic Epistemic Logic
2. Symbolic **Model Checking**
3. Factual Change
4. Symbolic Bisimulations

Model Checking – The Task

Given a model and a formula, does it hold in the model?

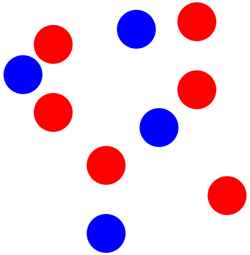
$$\mathcal{M}, w \models \varphi \quad \text{or} \quad \mathcal{M}, w \not\models \varphi$$

???

Limits of Explicit Model Checking

Set of possible worlds has to fit in memory.
For large models ($\sim$ 1000 worlds) this gets slow.
Runtime in seconds for n Muddy Children:

n	DEMO-S5
6	0.012
8	0.273
10	8.424
11	46.530
12	228.055



1. Dynamic Epistemic Logic
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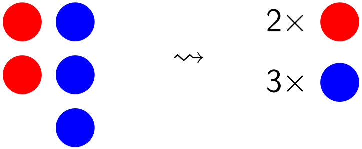
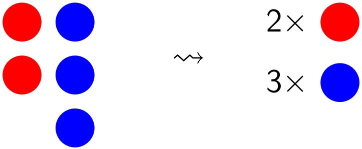
Symbolic Model Checking



- Can we represent models in a more compact way?
  . . . such that we can still interpret all formulas?

Symbolic Model Checking



- Can we represent models in a more compact way?
  . . . such that we can still interpret all formulas?

Yes! Well-developed and successful for temporal logics ATL, CTL, etc.

1. Represent $\mathcal{M} = (W, R_i, \mathsf{Val})$ symbolically: $\mathcal{F} = (V, \theta, O_i)$.
2. Translate DEL to equivalent *boolean formulas*.
3. Use Binary Decision Diagrams to speed it up.

Knowledge Structures

Knowledge Structures

$\mathcal{F} = (V, \theta, O_1, \cdots, O_n)$ where

V	*Vocabulary*	set of propositional variables
θ	*State Law*	boolean formula over V
$O_i \subseteq V$	*Observables*	propositions observable by i

The set of states is $\{s \subseteq V \mid s \vDash \theta\}$. Call $(\mathcal{F}, s)$ a scenario.

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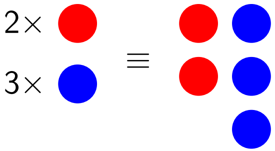
Symbolic Semantics

$\mathcal{F}, s \vDash K\varphi$ iff $s \cap O_i = s' \cap O_i$ implies $\mathcal{F}, s' \vDash \varphi$

From Knowledge Structures to Kripke Models

Theorem
For every knowledge structure there is an equivalent S5 Kripke Model and vice versa.

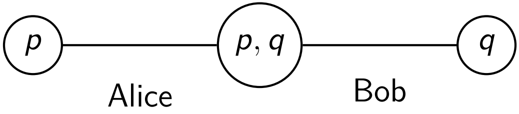


Example

The knowledge structure

$$\mathcal{F} = (V = \{p, q\}, \theta = p \lor q, O_{\text{Alice}} = \{p\}, O_{\text{Bob}} = \{q\})$$

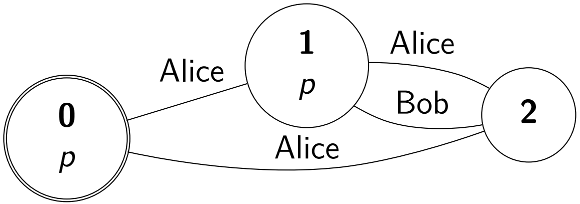
is equivalent to this Kripke model:



From Kripke Models to Knowledge Structures

(This is the tricky direction.)

Example



is equivalent to this knowledge structure:

$$(\ V = \{p, p_2\}, \ \theta = p_2 \to p, \ O_{\text{Alice}} = \varnothing, \ O_{\text{Bob}} = \{p_2\} \ )$$

with actual state: $\{p, p_2\}$

Everything is boolean!

Definition

Fix knowledge structure $\mathcal{F} = (V, \theta, O)$. Define *local boolean translation*:

- $\|p\|_{\mathcal{F}} := p$
- $\|\neg\varphi\|_{\mathcal{F}} := \neg\|\varphi\|_{\mathcal{F}}$
- $\|\varphi_1 \wedge \varphi_2\|_{\mathcal{F}} := \|\varphi_1\|_{\mathcal{F}} \wedge \|\varphi_2\|_{\mathcal{F}}$
- $\|K_i\varphi\|_{\mathcal{F}} := \forall(V \setminus O_i)(\theta \to \|\varphi\|_{\mathcal{F}})$

To evaluate public announcements:

$\mathcal{F}, s \vDash [\varphi]\psi$ iff $\mathcal{F}, s \vDash \varphi$ implies $(V, \theta \wedge \|\varphi\|_{\mathcal{F}}, O), s \vDash \psi$.

- $\|[!\varphi]\psi\|_{\mathcal{F}} := \|\varphi\|_{\mathcal{F}} \to \|\psi\|_{\mathcal{F}^{\varphi}}$

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Theorem

For all scenarios $(\mathcal{F}, s)$ and all formulas φ: $\mathcal{F}, s \vDash \varphi \iff s \vDash \|\varphi\|_{\mathcal{F}}$

Example: Symbolic Muddy Children I

$$\mathcal{F}_0 = \left(V = \{p_1, p_2, p_3\}, \theta_0 = \top, \begin{array}{l} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

"At least one of you is muddy."

Example: Symbolic Muddy Children I

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"At least one of you is muddy."

$$\mathcal{F}_1 = \left(V = \{p_1, p_2, p_3\}, \theta_1 = (p_1 \vee p_2 \vee p_3), \begin{array}{l} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

Example: Symbolic Muddy Children I

$$\mathcal{F}_0 = \left(V = \{p_1, p_2, p_3\}, \theta_0 = \top, \begin{array}{l} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

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"Do you know if you are muddy?" ... Nobody reacts.

$$\bigwedge_{i \in I} (\neg (K_i p_i \vee K_i \neg p_i))$$

Example: Symbolic Muddy Children II

$$\begin{aligned}
\|K_1 p_1\|_{\mathcal{F}_1} &= \forall(V \setminus O_1)(\theta_1 \to \|p_1\|_{\mathcal{F}_1}) \\
&= \forall p_1((p_1 \vee p_2 \vee p_3) \to p_1) \\
&= ((\top \vee p_2 \vee p_3) \to \top) \wedge ((\bot \vee p_2 \vee p_3) \to \bot) \\
&= \neg(p_2 \vee p_3)
\end{aligned}$$

$$\begin{aligned}
\|K_1 \neg p_1\|_{\mathcal{F}_1} &= \forall(V \setminus O_1)(\theta_1 \to \|\neg p_1\|_{\mathcal{F}_1}) \\
&= \forall p_1((p_1 \vee p_2 \vee p_3) \to \neg p_1) \\
&= ((\top \vee p_2 \vee p_3) \to \neg\top) \wedge ((\bot \vee p_2 \vee p_3) \to \neg\bot) \\
&= \bot
\end{aligned}$$

and analogous for $K_2 p_2$, $K_2 \neg p_2$, $K_3 p_3$ and $K_3 \neg p_3$

Example: Symbolic Muddy Children III

This announcement is equivalent to:

$$\| \bigwedge_{i \in I} (\neg (K_i p_i \vee K_i \neg p_i)) \|_{\mathcal{F}_1} \; = \; (p_2 \vee p_3) \wedge (p_1 \vee p_3) \wedge (p_1 \vee p_2)$$

"Noboy knows their own state."

is locally equivalent to

"At least two are muddy."

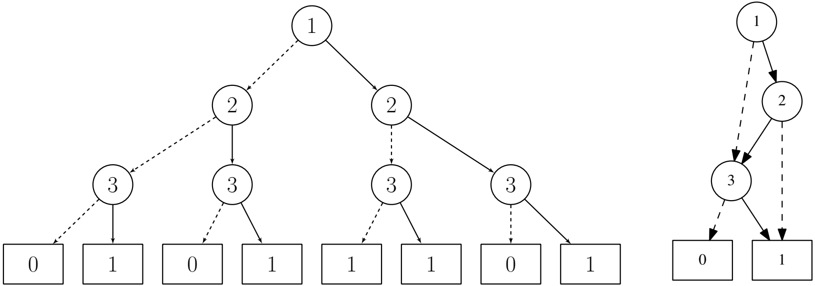
Results

Why translate DEL to boolean formulas?

Because computers are incredibly good at dealing with them!

n	DEMO-S5	SMCDEL
6	0.012	0.002
8	0.273	0.004
10	8.424	0.008
11	46.530	0.011
12	228.055	0.015
13	1215.474	0.019
20		0.078
40		0.777
60		2.563

Magic from 1986: Binary Decision Diagrams



(Read the classic Bryant 1986 for more details.)

BDD Magic I

How long do you need to compare these two formulas?

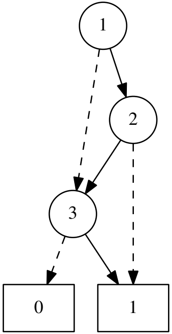
$$p_3 \lor \neg(p_1 \rightarrow p_2) \quad ??? \quad \neg(p_1 \land \neg p_2) \rightarrow p_3$$

BDD Magic I

How long do you need to compare these two formulas?

$$p_3 \vee \neg(p_1 \to p_2) \quad ??? \quad \neg(p_1 \wedge \neg p_2) \to p_3$$

Here ~~are~~ is their BDD~~s~~:



BDD Magic II

This was not an accident, BDDs are canonical.

Theorem:

$$\varphi \equiv \psi \quad \iff \quad \mathsf{BDD}(\varphi) = \mathsf{BDD}(\psi)$$

Equivalence checks are free and we have fast algorithms to compute BDD($\neg\varphi$), BDD($\varphi \wedge \psi$), BDD($\varphi \to \psi$) etc.

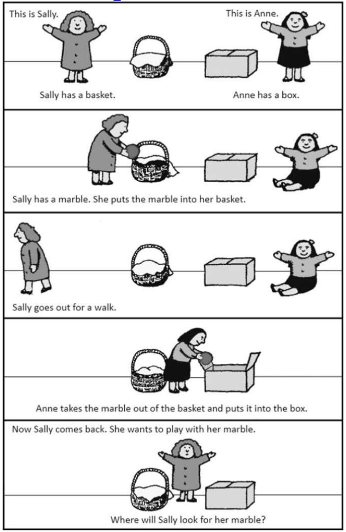
So far, so good . . .

This was S5 PAL, what about:

1. Non-S5 relations for belief?

2. All action models, with factual change?

Motivating Example: Sally-Anne



To model this we need non-S5 action models with factual change!

Extension to non-S5

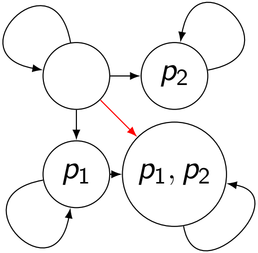
- $O_i \subseteq P$ always defines equivalence relations!
- How can we describe other relations?

Extension to non-S5

- $O_i \subseteq P$ always defines equivalence relations!
- How can we describe other relations?

- Copy P to describe reachability (Gorogiannis and Ryan 2002)
- Fresh variables $V' := \{p' \mid p \in V\}$ and formulas $\mathcal{L}(V \cup V')$
- Actually: use BDD of boolean formula describing the relation

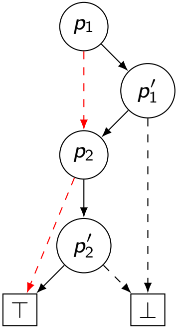
Extension to non-S5: From relations to BDDs



Relation R

$$
\begin{aligned}
&(\neg p_1 \wedge \neg p_2 \wedge \neg p_1' \wedge \neg p_2') \\
&\vee (\neg p_1 \wedge \neg p_2 \wedge \neg p_1' \wedge p_2') \\
&\vee (\neg p_1 \wedge \neg p_2 \wedge p_1' \wedge \neg p_2') \\
&\vee (\neg p_1 \wedge \neg p_2 \wedge p_1' \wedge p_2') \\
&\vee (\neg p_1 \wedge p_2 \wedge \neg p_1' \wedge p_2') \\
&\vee (p_1 \wedge \neg p_2 \wedge p_1' \wedge p_2') \\
&\vee (p_1 \wedge \neg p_2 \wedge p_1' \wedge \neg p_2') \\
&\vee (p_1 \wedge p_2 \wedge p_1' \wedge p_2')
\end{aligned}
$$

Formula $\Phi(R)$.



BDD of $\Phi(R)$

Extension to non-S5: Belief Structures

$\mathcal{F} = (V, \theta, \Omega_1, \cdots, \Omega_n)$ where

- V — *Vocabulary* — propositional variables
- θ — *State Law* — boolean formula over V
- $\Omega_i \in \mathcal{L}(V \cup V')$ — *Observables* — encoded relation for i

The equivalent Kripke model is given by:

$$R_i xy \;:\Longleftrightarrow\; (x \cup y') \models \Omega_i$$

New translation for modalities:

$$\|\Box_i \psi\|_{\mathcal{F}} := \forall V'(\theta' \rightarrow (\Omega_i \rightarrow (\|\varphi\|_{\mathcal{F}})'))$$

(Some redundancy between θ and Ω_i here...)

The return of the postcondition

1. Dynamic Epistemic Logic
2. Symbolic Model Checking
3. **Factual Change**
4. Symbolic Bisimulations

Transformers

Definition

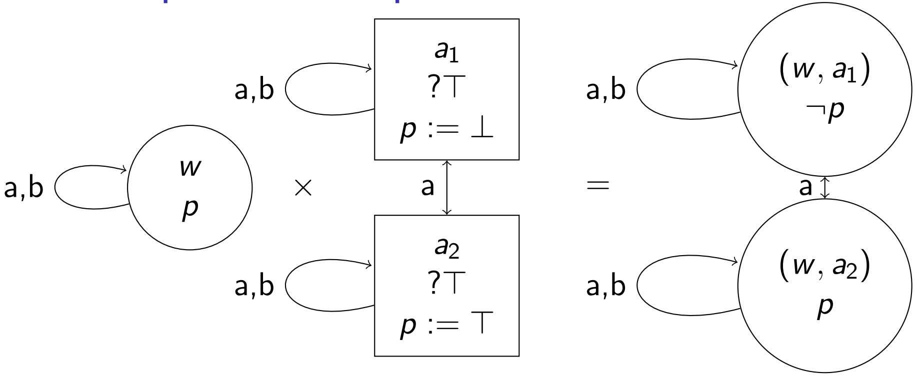
A *transformer* for V is a tuple $\mathcal{X} = (V^+, \theta^+, V_-, \theta_-, \Omega^+)$ where

- V^+ is a set of fresh atomic propositions s.t. $V \cap V^+ = \varnothing$
- θ^+ is a possibly epistemic formula from $\mathcal{L}(V \cup V^+)$
- $V_- \subseteq V$ is the *modified subset* of the original vocabulary
- $\theta_- : V_- \to \mathcal{L}_B(V \cup V^+)$ encodes postconditions
- $\Omega_i^+ \in \mathcal{L}_B(V^+ \cup V^{+\prime})$ describe observations for each i

to transform $\mathcal{F} = (V, \theta, \Omega_i)$, let $\mathcal{F} \times \mathcal{X} := (V^{\text{new}}, \theta^{\text{new}}, \Omega_i^{\text{new}})$ where

1. $V^{\text{new}} \;:=\; V \cup V^+ \cup V_-^\circ$
2. $\theta^{\text{new}} \;:=\; \left[V_-/V_-^\circ\right](\theta \wedge \|\theta^+\|_{\mathcal{F}}) \;\wedge\; \bigwedge_{q \in V_-} \left(q \leftrightarrow \left[V_-/V_-^\circ\right](\theta_-(q))\right)$
3. $\Omega_i^{\text{new}} \;:=\; \left(\left[V_-/V_-^\circ\right]\left[(V_-)'/(V_-^\circ)'\right]\Omega_i\right) \wedge \Omega_i^+$

Simple Example: Coin Flip hidden from a



$$
\begin{array}{llll}
 & (V = \{p\}, & \theta = p, & \Omega_a = \top, \quad \Omega_b = \top) \\
\times & (V^+ = \{q\}, & \theta^+ = \top, & \Omega_a^+ = \top, \quad \Omega_b^+ = q \leftrightarrow q') \\
 & \;V_- = \{p\}, & \theta_-(p) := q, & \\
= & (V = \{p, q, p^\circ\} & \theta = p^\circ \wedge (p \leftrightarrow q), & \Omega_a = \top \quad \Omega_b = q \leftrightarrow q')
\end{array}
$$

Symbolic Equivalence

1. Dynamic Epistemic Logic
2. Symbolic Model Checking
3. Factual Change
4. **Symbolic Bisimulations**

When are two knowledge structures equivalent?

What is a symbolic representation for bisimulations?

Note: Standard bisimulation is a three-variable FOL condition.

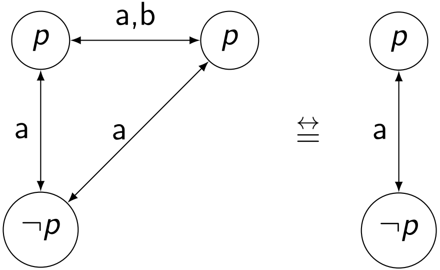
Bipropulations: Propositionally Encoded Bisimulations

Definition

Suppose $\mathcal{F}_1 = (V_1, \theta_1, O_1^i)$, $\mathcal{F}_2 = (V_2, \theta_2, O_2^i)$ and let $V := V_1 \cap V_2$. A boolean formula $\beta \in \mathcal{L}_B(V \cup V^*)$ is a *bipropulation* between $\mathcal{F}_1$ and $\mathcal{F}_2$ iff:

- $\beta \to \bigwedge_{p \in V}(p \leftrightarrow p^*)$ is valid
- Suppose $s_1 \cup (s_2^*) \models \beta$, and $O_1^i \cap s_1 = O_1^i \cap t_1$ in F_1.
  Then there is a t_2 such that $t_1 \cup (t_2^*) \models \beta$ and $O_2^i \cap s_2 = O_2^i \cap t_2$ in F_2.
- vice versa

Bipropulation Example



$(V = \{p, q\}, \theta = p \vee q, O_a = \varnothing, O_b = \{p\})$

$(V = \{p\}, \theta = \top, O_a = \varnothing, O_b = \{p\})$

$$\beta := p \leftrightarrow p^*$$

Bipropulations

Lemma

The following are equivalent:

- β is a bipropulation
- β encodes a bisimiluation between the encoded S5 Kripke models
- the following boolean formulas are tautologies:

$$\beta \to \bigwedge_{p \in V} (p \leftrightarrow p^*)$$

$$\forall (V \cup V^*) : \beta \to \bigwedge_{i} \Big(\forall (V \setminus O_1^i) : \exists (V^* \setminus (O_2^i)^*) : \beta' \Big)$$

Symbolic Van Benthem Theorem

Two knowledge structures are equivalent iff there is a bipropulation between them (restricted to a certain vocabulary).

Non-S5 Bipropulations

Definition

Suppose $\mathcal{F}_1 = (V_1, \theta, \Omega_1^i)$, $\mathcal{F}_2 = (V_1, \theta, \Omega_2^i)$ and let $V := V_1 \cap V_2$.
A boolean formula $\beta \in \mathcal{L}(V \cup V^*)$ is called a *bipropulation* iff:

- $\beta \to \bigwedge_{p \in V}(p \leftrightarrow p^*)$ is a tautology (i.e. its BDD is equal to $\top$)
- Take any s_1 and s_2 such that $s_1 \cup (s_2^*) \vDash \beta$, any agents i and any t_1 such that $s_1 \cup t_1' \vDash \Omega_1^i$ in F_1.
  Then there is a t_2 such that $t_1 \cup (t_2^*) \vDash \beta$ and $s_2 \cup t_2' \vDash \Omega_2^i$ in F_2.
- vice versa

As a boolean formula, with four copies of variables for "forth":

$$\forall (V \cup V^*) : \beta \to \bigwedge_i \left(\forall V' : \Omega_i^1 \to \exists V^{*\prime} : \beta' \wedge (\Omega_i^2)^*)\right)$$

Summary

- representation matters!
- *symbolic* model checking DEL gives a big speed-up
- *knowledge/belief structures* encode Kripke models
- "double vocabulary trick"
  - encode non-S5 relation
  - *transformers*: modular approach for ontic & epistemic actions
  - *propulations*: encode bisimulations using propositions

Summary

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- *symbolic* model checking DEL gives a big speed-up
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Future Work

- Mini/Opti-mization after update
- Find big examples and benchmark
- Maybe limit postconditions to $\top$ and $\bot$ (van Ditmarsch and Kooi 2008)
- Comparison with (Charrier and Schwarzentruber 2017)

papers & software $\Rightarrow$ `https://malv.in`

Optimization Techniques

Minimization Lemma

Suppose $\mathcal{F}$ has vocabulary $V \cup \{p\}$ and $p \notin V$ is determined by the state law (i.e. $\theta \rightarrow p$ or $\theta \rightarrow \neg p$ is a tautology). Then we can compute a smaller $\mathcal{L}(V)$-equivalent structure by removing p from state law and observations.

Example

The coin flip result

$$(V = \{p, q, p^\circ\},\ \theta = p^\circ \wedge (p \leftrightarrow q),\ \Omega_a = p^\circ \leftrightarrow p^{\circ\prime},\ \Omega_b = (p^\circ \leftrightarrow p^{\circ\prime}) \wedge (q \leftrightarrow q'))$$

is $\mathcal{L}(\{p, q\})$-equivalent to:

$$(V = \{p, q\},\ \theta = p \leftrightarrow q,\ \Omega_a = \top,\ \Omega_b = q \leftrightarrow q')$$

References with Links

First publication: (van Benthem et al. 2015)
https://doi.org/bzb6

Non-S5 and action models: (van Benthem et al. to appear)
http://homepages.cwi.nl/~jve/papers/16/pdfs/2016-05-23-del-bdd-lori-journal.pdf

Adding factual change: (Gattinger 2017)
https://w4eg.de/malvin/illc/2017-07-symbolicfactualchange.pdf

Ongoing implementation: (Gattinger 2015)
https://github.com/jrclogic/smcdel

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