Towards Symbolic Factual Change in Dynamic Epistemic Logic

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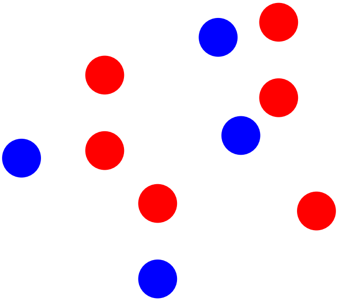
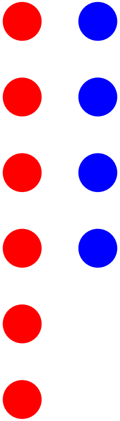


Are there more red or more blue points?



Are there more red or more blue points?

$6\times$ ● $4\times$ ●

Are there more red or more blue points?

Representation matters!

1. **Dynamic Epistemic Logic**
2. Symbolic Model Checking
3. Factual Change

Epistemic Logic

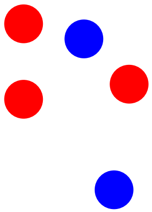
Syntax

$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi$

Kripke Models

$\mathcal{M} = (W, R_i, \mathrm{Val})$ where

- W set of worlds
- $R_i \subseteq W \times W$ indistinguishability relation
- $\mathrm{Val} : W \to \mathcal{P}(P)$ valuation function



Semantics

$\mathcal{M}, w \models K_i\varphi$ iff wR_iv implies $\mathcal{M}, v \models \varphi$

Dynamic Epistemic Logic: Action Models

Action Models

$\mathcal{A} = (A, R, \text{pre}, \text{post})$ where

- A set of atomic events
- $R_i \subseteq A \times A$ indistinguishability relation
- $\text{pre} : A \to \mathcal{L}$ precondition function
- $\text{post} : A \to P \to \mathcal{L}$ postcondition function

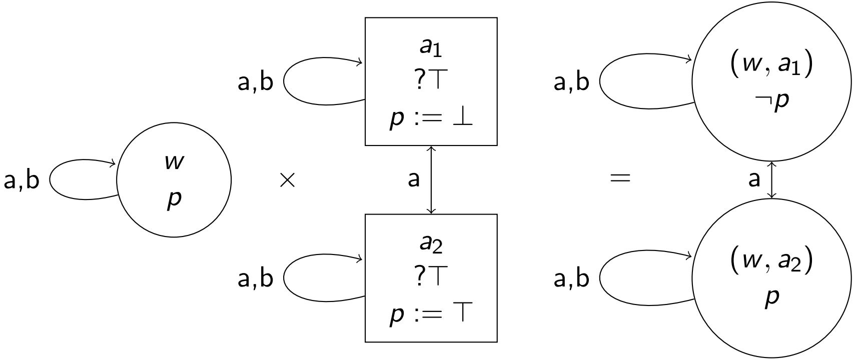
Product Update

$\mathcal{M} \times \mathcal{A} := (W^{\text{new}}, \mathcal{R}_i^{\text{new}}, \text{Val}^{\text{new}})$ where

- $W^{\text{new}} := \{(w, a) \in W \times A \mid \mathcal{M}, w \vDash \text{pre}(a)\}$
- $\mathcal{R}_i^{\text{new}} := \{((w, a), (v, b)) \mid \mathcal{R}_i wv \text{ and } R_i ab\}$
- $\text{Val}^{\text{new}}((w, a)) := \{p \in V \mid \mathcal{M}, w \vDash \text{post}_a(p)\}$

(Baltag, Moss, and Solecki 1998,Benthem, Eijck, and Kooi (2006))

DEL Example: Coin Flip hidden from a



1. Dynamic Epistemic Logic
2. Symbolic **Model Checking**
3. Factual Change

Model Checking – The Task

Given a model and a formula, does it hold in the model?

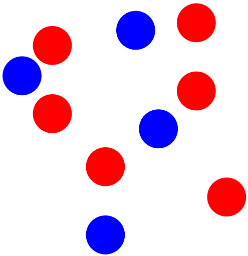
$$\mathcal{M}, w \models \varphi \quad \text{or} \quad \mathcal{M}, w \not\models \varphi$$

???

Limits of Explicit Model Checking

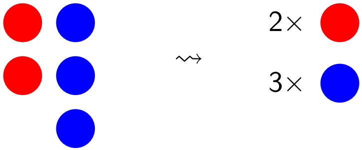
Set of possible worlds has to fit in memory.
For large models ($\sim$ 1000 worlds) this gets slow.
Runtime in seconds for n Muddy Children:

n	DEMO-S5
6	0.012
8	0.273
10	8.424
11	46.530
12	228.055



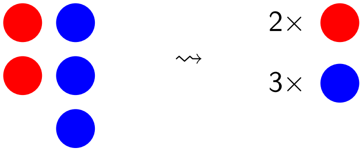
1. Dynamic Epistemic Logic
2. **Symbolic** Model Checking
3. Factual Change

Symbolic Model Checking



- Can we represent models in a more compact way?
  … such that we can still interpret all formulas?

Symbolic Model Checking



- Can we represent models in a more compact way?
  ... such that we can still interpret all formulas?

Yes!

1. Represent $\mathcal{M} = (W, R_i, \text{Val})$ symbolically: $\mathcal{F} = (V, \theta, O_i)$.
2. Translate DEL to equivalent *boolean formulas*.
3. Use Binary Decision Diagrams to speed it up.

Knowledge Structures

Knowledge Structures

$\mathcal{F} = (V, \theta, O_1, \cdots, O_n)$ where

- V *Vocabulary* set of propositional variables
- θ *State Law* boolean formula over V
- $O_i \subseteq V$ *Observables* propositions observable by i

The set of states is $\{s \subseteq V \mid s \vDash \theta\}$. Call $(\mathcal{F}, s)$ a scenario.

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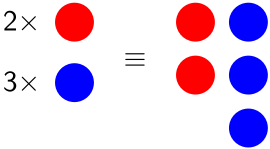
Symbolic Semantics

$\mathcal{F}, s \vDash K\varphi$ iff $s \cap O_i = s' \cap O_i$ implies $\mathcal{F}, s' \vDash \varphi$

From Knowledge Structures to Kripke Models

Theorem
For every knowledge structure there is an equivalent S5 Kripke Model and vice versa.

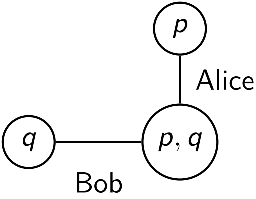


Example

The knowledge structure

$$\mathcal{F} = (V = \{p, q\}, \theta = p \lor q, O_{\text{Alice}} = \{p\}, O_{\text{Bob}} = \{q\})$$

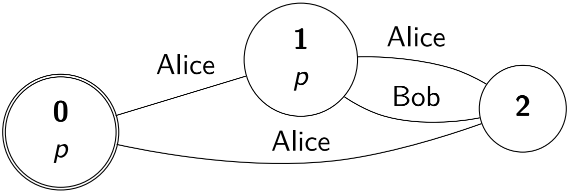
is equivalent to this Kripke model:



From Kripke Models to Knowledge Structures

(This is the tricky direction.)

Example



is equivalent to this knowledge structure:

$$(\ V = \{p, p_2\}, \ \theta = p_2 \rightarrow p, \ O_{\text{Alice}} = \varnothing, \ O_{\text{Bob}} = \{p_2\} \ )$$

with actual state: $\{p, p_2\}$

Everything is boolean!

Definition

Fix a knowledge structure $\mathcal{F} = (V, \theta, O_1, \cdots, O_n)$.
We define a *local boolean translation* $\| \cdot \|_{\mathcal{F}}$:

- $\|p\|_{\mathcal{F}} := p$
- $\|\neg\varphi\|_{\mathcal{F}} := \neg\|\varphi\|_{\mathcal{F}}$
- $\|\varphi_1 \wedge \varphi_2\|_{\mathcal{F}} := \|\varphi_1\|_{\mathcal{F}} \wedge \|\varphi_2\|_{\mathcal{F}}$
- $\|K_i\varphi\|_{\mathcal{F}} := \forall(V \setminus O_i)(\theta \rightarrow \|\varphi\|_{\mathcal{F}})$
- $\|[!\varphi]\psi\|_{\mathcal{F}} := \|\varphi\|_{\mathcal{F}} \rightarrow \|\psi\|_{\mathcal{F}^\varphi}$

Everything is boolean!

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- $\|K_i\varphi\|_{\mathcal{F}} := \forall(V \setminus O_i)(\theta \to \|\varphi\|_{\mathcal{F}})$
- $\|[!\varphi]\psi\|_{\mathcal{F}} := \|\varphi\|_{\mathcal{F}} \to \|\psi\|_{\mathcal{F}^\varphi}$

Theorem

For all scenarios $(\mathcal{F}, s)$ and all formulas φ:
$\mathcal{F}, s \vDash \varphi \iff s \vDash \|\varphi\|_{\mathcal{F}}$

Example: Symbolic Muddy Children I

$$\mathcal{F}_0 = \left(V = \{p_1, p_2, p_3\}, \theta_0 = \top, \begin{array}{l} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

"At least one of you is muddy."

Example: Symbolic Muddy Children I

$$\mathcal{F}_0 = \left(V = \{p_1, p_2, p_3\}, \theta_0 = \top, \begin{array}{c} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

"At least one of you is muddy."

$$\mathcal{F}_1 = \left(V = \{p_1, p_2, p_3\}, \theta_1 = (p_1 \vee p_2 \vee p_3), \begin{array}{c} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

Example: Symbolic Muddy Children I

$$\mathcal{F}_0 = \left(V = \{p_1, p_2, p_3\}, \theta_0 = \top, \begin{array}{l} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

"At least one of you is muddy."

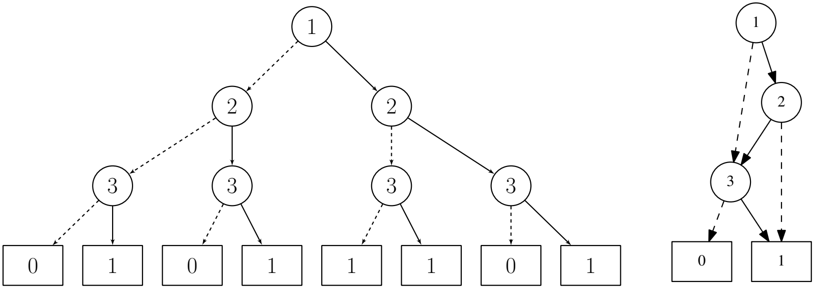
$$\mathcal{F}_1 = \left(V = \{p_1, p_2, p_3\}, \theta_1 = (p_1 \vee p_2 \vee p_3), \begin{array}{l} O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right)$$

"Do you know if you are muddy?" ... Nobody reacts.

This announcement is equivalent to:

$$\| \bigwedge_{i \in I} (\neg (K_i p_i \vee K_i \neg p_i)) \|_{\mathcal{F}_1} \;=\; (p_2 \vee p_3) \wedge (p_1 \vee p_3) \wedge (p_1 \vee p_2)$$

Magic from 1986: Binary Decision Diagrams



(Read the classic Bryant 1986 for more details.)

BDD Magic I

How long do you need to compare these two formulas?

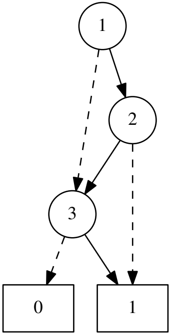
$$p_3 \vee \neg(p_1 \to p_2) \quad \equiv \quad \neg(p_1 \wedge \neg p_2) \to p_3$$

BDD Magic I

How long do you need to compare these two formulas?

$$p_3 \lor \neg(p_1 \rightarrow p_2) \quad \equiv \quad \neg(p_1 \land \neg p_2) \rightarrow p_3$$

Here ~~are~~ is their BDD~~s~~:



BDD Magic II

This was not an accident, BDDs are canonical.

Theorem:

$$\varphi \equiv \psi \quad \iff \quad \mathsf{BDD}(\varphi) = \mathsf{BDD}(\psi)$$

Equivalence checks are free and we have fast algorithms to compute $\mathsf{BDD}(\neg\varphi)$, $\mathsf{BDD}(\varphi \land \psi)$, $\mathsf{BDD}(\varphi \to \psi)$ etc.

Results

Why translate DEL to boolean formulas?

Because computers are incredibly good at dealing with them!

n	DEMO-S5	SMCDEL
6	0.012	0.002
8	0.273	0.004
10	8.424	0.008
11	46.530	0.011
12	228.055	0.015
13	1215.474	0.019
20		0.078
40		0.777
60		2.563

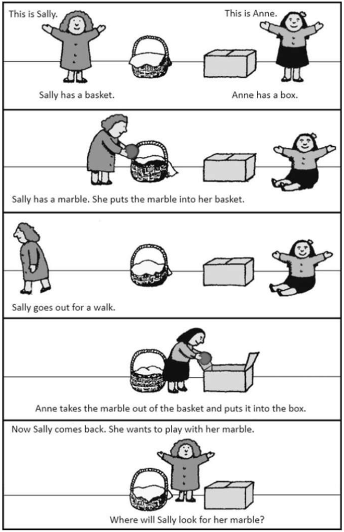
So far, so good . . .

This was S5 PAL, what about:

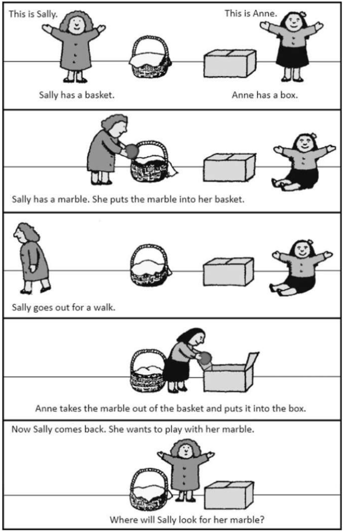
1. Non-S5 relations for belief?

2. Action Models with factual change?

Motivating Example: Sally-Anne



Motivating Example: Sally-Anne



To model this we need non-S5 action models with factual change!

Extension to non-S5

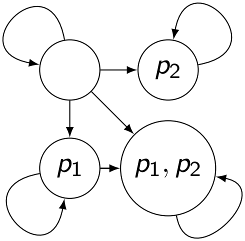
- $O_i \subseteq P$ always defines equivalence relations!
- How can we describe other relations?

Extension to non-S5

- $O_i \subseteq P$ always defines equivalence relations!
- How can we describe other relations?

- Copy P to describe reachability (Gorogiannis and Ryan 2002)
- Fresh variables $V' := \{p' \mid p \in V\}$ and formulas $\mathcal{L}(V \cup V')$
- Actually: use BDD of boolean formula describing the relation

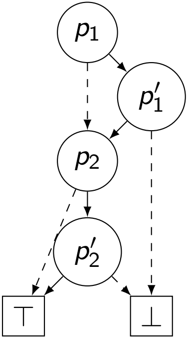
Extension to non-S5: From relations to BDDs



Relation R

$$
\begin{aligned}
&(\neg p_1 \land \neg p_2 \land \neg p_1' \land \neg p_2') \\
\lor\ &(\neg p_1 \land \neg p_2 \land \neg p_1' \land p_2') \\
\lor\ &(\neg p_1 \land \neg p_2 \land p_1' \land \neg p_2') \\
\lor\ &(\neg p_1 \land \neg p_2 \land p_1' \land p_2') \\
\lor\ &(\neg p_1 \land p_2 \land \neg p_1' \land p_2') \\
\lor\ &(p_1 \land \neg p_2 \land p_1' \land p_2') \\
\lor\ &(p_1 \land \neg p_2 \land p_1' \land \neg p_2') \\
\lor\ &(p_1 \land p_2 \land p_1' \land p_2')
\end{aligned}
$$

Formula $\Phi(R)$.



BDD of $\Phi(R)$

Extension to non-S5: Belief Structures

$\mathcal{F} = (V, \theta, \Omega_1, \cdots, \Omega_n)$ where

▶ V	*Vocabulary*	propositional variables
▶ θ	*State Law*	boolean formula over V
▶ $\Omega_i \in \mathcal{L}(V \cup V')$	*Observables*	encoded relation for i

The equivalent Kripke model is given by:

$$R_i xy \;\; :\Longleftrightarrow \;\; (x \cup y') \vDash \Omega_i$$

New translation for modalities:

$$\|\Box_i \psi\|_{\mathcal{F}} := \forall V'(\theta' \rightarrow (\Omega_i \rightarrow (\|\varphi\|_{\mathcal{F}})'))$$

The return of the postcondition

1. Dynamic Epistemic Logic
2. Symbolic Model Checking
3. **Factual Change**

Transformers

Definition

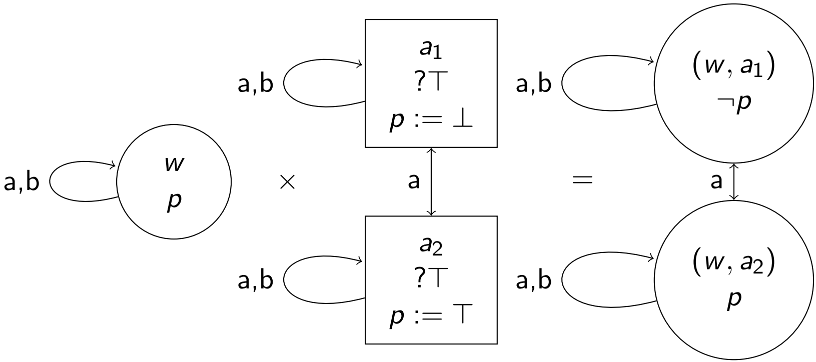
A *transformer* for V is a tuple $\mathcal{X} = (V^+, \theta^+, V_-, \theta_-, \Omega^+)$ where

- V^+ is a set of fresh atomic propositions s.t. $V \cap V^+ = \varnothing$
- θ^+ is a possibly epistemic formula from $\mathcal{L}(V \cup V^+)$
- $V_- \subseteq V$ is the *modified subset* of the original vocabulary
- $\theta_- : V_- \to \mathcal{L}_B(V \cup V^+)$ encodes postconditions
- $\Omega_i^+ \in \mathcal{L}_B(V^+ \cup V^{+\prime})$ describe observations for each i

to transform $\mathcal{F} = (V, \theta, \Omega_i)$, let $\mathcal{F} \times \mathcal{X} := (V^{\text{new}}, \theta^{\text{new}}, \Omega_i^{\text{new}})$ where

1. $V^{\text{new}} \;:=\; V \cup V^+ \cup V_-^\circ$
2. $\theta^{\text{new}} \;:=$
   $\left[V_-/V_-^\circ\right](\theta \wedge \|\theta^+\|_{\mathcal{F}}) \;\wedge\; \bigwedge_{q \in V^-} (q \leftrightarrow \left[V_-/V_-^\circ\right](\theta_-(q)))$
3. $\Omega_i^{\text{new}} \;:=\; (\left[V_-/V_-^\circ\right]\left[(V_-)'/(V_-^\circ)'\right]\Omega_i) \wedge \Omega_i^+$

Simple Example: Coin Flip hidden from a



$$
\begin{array}{lllll}
 & (V = \{p\}, & \theta = p, & \Omega_a = \top, & \Omega_b = \top) \\
\times & (V^+ = \{q\} & \theta^+ = \top, & \Omega_a^+ = \top, & \Omega_b^+ = q \leftrightarrow q') \\
 & \ V_- = \{p\}, & \theta_-(p) := q, & & \\
= & (V = \{p, q, p^\circ\} & \theta = p^\circ \wedge (p \leftrightarrow q), & \Omega_a = \top & \Omega_b = q \leftrightarrow q')
\end{array}
$$

Monster Example: Symbolic Sally-Anne I

- Sally is in the room: p, Marble is in the box: t
- Question after the actions: Does Sally believe that the marble is in the box: $\Box_s t$?

Monster Example: Symbolic Sally-Anne I

- Sally is in the room: p, Marble is in the box: t
- Question after the actions: Does Sally believe that the marble is in the box: $\Box_s t$?

Initial structure: $((\{p, t\}, (p \wedge \neg t), \top, \top), p)$

$\mathcal{X}_1$: Sally puts the marble in the basket:
$((\varnothing, \top, \{t\}, \theta_-(t) = \top, \top, \top), \varnothing)$.

$\mathcal{X}_2$: Sally leaves: $((\varnothing, \top, \{p\}, \theta_-(p) = \bot, \top, \top), \varnothing)$.

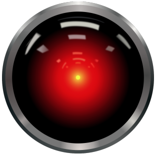
$\mathcal{X}_3$: Anne puts the marble in the box, not observed by Sally:
$((\{q\}, \top, \{t\}, \theta_-(t) = (\neg q \rightarrow t) \wedge (q \rightarrow \bot), \neg q', q \leftrightarrow q'), \{q\})$.

$\mathcal{X}_4$: Sally comes back: $((\varnothing, \top, \{p\}, \theta_-(p) = \top, \top, \top), \varnothing)$.

The slide we will skip

$$
\begin{array}{ll}
 & ((\{p,t\},(p\wedge\neg t),\top,\top),p)\\
\times & ((\varnothing,\top,\{t\},\theta_-(t)=\top,\top,\top),\varnothing)\\
= & ((\{p,t,t^\circ\},(p\wedge\neg t^\circ)\wedge t,\top,\top),\{p,t\})\\
 & \\
\times & ((\varnothing,\top,\{p\},\theta_-(p)=\bot,\top,\top),\varnothing)\\
= & ((\{p,t,t^\circ,p^\circ\},(p^\circ\wedge\neg t^\circ)\wedge t\wedge\neg p,\top,\top),\{t,p^\circ\})\\
\equiv_V & ((\{p,t\},t\wedge\neg p,\top,\top),\{t\})\\
 & \\
\times & ((\{q\},\top,\{t\},\theta_-(t)=(\neg q\rightarrow t)\wedge(q\rightarrow\bot),\neg q',q\leftrightarrow q'),\{q\})\\
= & ((\{p,t,q,t^\circ\},t^\circ\wedge\neg p\wedge(t\leftrightarrow((\neg q\rightarrow t^\circ)\wedge(q\rightarrow\bot))),\neg q',q\leftrightarrow q'),\{q\\
= & ((\{p,t,q,t^\circ\},t^\circ\wedge\neg p\wedge(t\leftrightarrow\neg q),\neg q',q\leftrightarrow q'),\{q\})\\
\equiv_V & ((\{p,t,q\},\neg p\wedge(t\leftrightarrow\neg q),\neg q',q\leftrightarrow q'),\{q\})\\
 & \\
\times & ((\varnothing,\top,\{p\},\theta_-(p)=\top,\top,\top),\varnothing)\\
= & ((\{p,t,q,p^\circ\},\neg p^\circ\wedge(t\leftrightarrow\neg q)\wedge p,\neg q',q\leftrightarrow q'),\{p,q\})\\
\equiv_V & ((\{p,t,q\},(t\leftrightarrow\neg q)\wedge p,\neg q',q\leftrightarrow q'),\{p,q\})
\end{array}
$$

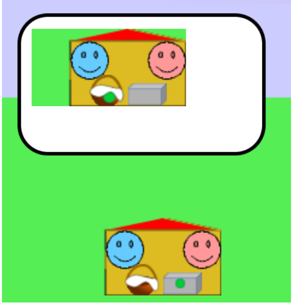
Everything on the previous scary slide are boolean operations, so we blindly trust a computer to deal with it.



Monster Example: Symbolic Sally-Anne IIII

In the last scene Sally believes the marble is in the basket:

$$
\begin{aligned}
& \{p, q\} \models \Box_S t \\
\iff \quad & \{p, q\} \models \forall V'(\theta' \to (\Omega_S \to t')) \\
\iff \quad & \{p, q\} \models \forall \{p', t', q'\}((t' \leftrightarrow \neg q') \wedge p' \to (\neg q' \to t')) \\
\iff \quad & \{p, q\} \models \top
\end{aligned}
$$



(François Schwarzentruber: *Hintikka's world*)

Summary

- representation matters!
- *symbolic* model checking DEL gives a big speed-up
- *knowledge/belief structures* encode Kripke models
- *transformers* provide a modular approach for ontic and epistemic actions

Future Work

- Comparison with (Charrier and Schwarzentruber 2017)
- Find big examples and benchmark!
- Limit postconditions to $\top$ and $\bot$?

`github.com/jrclogic/SMCDEL`

`malvin@w4eg.eu`

Bonus Slide: Comparison with Mental Programs / Succinct DEL

Similar approach in (Charrier and Schwarzentruber 2017). *Mental programs* describe which change is allowed: $q \mapsto \top, p \mapsto q, \dots$

- Observational BDDs and mental programs are dual in memory consumption:

Relation	BDD	Mental Program
Identity	$2 \cdot \lvert V \rvert$	1
Total	1	$2 \cdot \lvert V \rvert$

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