Towards Symbolic Factual Change in Dynamic Epistemic Logic

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ILLC, Amsterdam

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Toulouse
Are there more red or more blue points?
Are there more red or more blue points?
Are there more red or more blue points?
Representation matters!
1. Dynamic Epistemic Logic
2. Symbolic Model Checking
3. Factual Change
Epistemic Logic

Syntax
\[ \varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid K_i \varphi \]

Kripke Models
\[ \mathcal{M} = (W, R_i, \text{Val}) \] where
- \( W \) set of worlds
- \( R_i \subseteq W \times W \) indistinguishability relation
- \( \text{Val} : W \rightarrow \mathcal{P}(\mathcal{P}) \) valuation function

Semantics
\[ \mathcal{M}, w \models K_i \varphi \text{ iff } wR_i v \text{ implies } \mathcal{M}, v \models \varphi \]
Dynamic Epistemic Logic: Action Models

Action Models

\[ A = (A, R, \text{pre}, \text{post}) \]

- \( A \): set of atomic events
- \( R \subseteq A \times A \): indistinguishability relation
- \( \text{pre} : A \rightarrow \mathcal{L} \): precondition function
- \( \text{post} : A \rightarrow P \rightarrow \mathcal{L} \): postcondition function

Product Update

\[ \mathcal{M} \times A := (W^{\text{new}}, R_i^{\text{new}}, \text{Val}^{\text{new}}) \]

- \( W^{\text{new}} := \{ (w, a) \in W \times A \mid \mathcal{M}, w \models \text{pre}(a) \} \)
- \( R_i^{\text{new}} := \{ ((w, a), (v, b)) \mid R_i w v \text{ and } R_i a b \} \)
- \( \text{Val}^{\text{new}}((w, a)) := \{ p \in V \mid \mathcal{M}, w \models \text{post}_a(p) \} \)

(Baltag, Moss, and Solecki 1998, Benthem, Eijck, and Kooi (2006))
DEL Example: Coin Flip hidden from a

```
\begin{align*}
(a, b) \times (w, a_1) & \Rightarrow (w, a_1) \\
(a, b) \times (w, a_2) & \Rightarrow (w, a_2)
\end{align*}
```
1. Dynamic Epistemic Logic
2. Symbolic **Model Checking**
3. Factual Change
Model Checking – The Task

Given a model and a formula, does it hold in the model?

\[ M, w \models \varphi \quad \text{or} \quad M, w \not\models \varphi \]

???
Set of possible worlds has to fit in memory. For large models (~ 1000 worlds) this gets slow. Runtime in seconds for \( n \) Muddy Children:

<table>
<thead>
<tr>
<th>( n )</th>
<th>DEMO-S5</th>
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<tbody>
<tr>
<td>6</td>
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1. Dynamic Epistemic Logic
2. **Symbolic** Model Checking
3. Factual Change
Symbolic Model Checking

Can we represent models in a more compact way? 
... such that we can still interpret all formulas?
Symbolic Model Checking

Can we represent models in a more compact way? ... such that we can still interpret all formulas?

Yes!

1. Represent $\mathcal{M} = (W, R_i, \text{Val})$ symbolically: $\mathcal{F} = (V, \theta, O_i)$.
2. Translate DEL to equivalent boolean formulas.
3. Use Binary Decision Diagrams to speed it up.
Knowledge Structures

\[ \mathcal{F} = (V, \theta, O_1, \cdots, O_n) \]

- **V** \textit{Vocabulary} \quad set of propositional variables
- **\theta** \textit{State Law} \quad boolean formula over \( V \)
- **\( O_i \subseteq V \)** \textit{Observables} \quad propositions observable by \( i \)

The set of states is \( \{ s \subseteq V \mid s \models \theta \} \). Call \((\mathcal{F}, s)\) a scenario.
Knowledge Structures

$\mathcal{F} = (V, \theta, O_1, \cdots, O_n)$ where

- $V$ Vocabulary set of propositional variables
- $\theta$ State Law boolean formula over $V$
- $O_i \subseteq V$ Observables propositions observable by $i$

The set of states is $\{s \subseteq V \mid s \models \theta\}$. Call $(\mathcal{F}, s)$ a scenario.

Symbolic Semantics

$\mathcal{F}, s \models K \varphi$ iff $s \cap O_i = s' \cap O_i$ implies $\mathcal{F}, s' \models \varphi$
From Knowledge Structures to Kripke Models

**Theorem**
For every knowledge structure there is an equivalent S5 Kripke Model and vice versa.

**Example**
The knowledge structure

\[ \mathcal{F} = (V = \{p, q\}, \theta = p \lor q, O_{Alice} = \{p\}, O_{Bob} = \{q\}) \]

is equivalent to this Kripke model:

\[ \equiv \]

2
3

\[ p \]

\[ q \]

\[ p, q \]

Alice
Bob
From Kripke Models to Knowledge Structures

(This is the tricky direction.)

Example

is equivalent to this knowledge structure:

\[( V = \{ p, p_2 \}, \ \theta = p_2 \rightarrow p, \ O_{\text{Alice}} = \emptyset, \ O_{\text{Bob}} = \{ p_2 \} ) \]

with actual state: \( \{ p, p_2 \} \)
Everything is boolean!

**Definition**

Fix a knowledge structure $\mathcal{F} = (V, \theta, O_1, \cdots, O_n)$. We define a *local boolean translation* $\| \cdot \|_\mathcal{F}$:

- $\| p \|_\mathcal{F} \coloneqq p$
- $\| \neg \varphi \|_\mathcal{F} \coloneqq \neg \| \varphi \|_\mathcal{F}$
- $\| \varphi_1 \land \varphi_2 \|_\mathcal{F} \coloneqq \| \varphi_1 \|_\mathcal{F} \land \| \varphi_2 \|_\mathcal{F}$
- $\| K_i \varphi \|_\mathcal{F} \coloneqq \forall (V \setminus O_i) (\theta \to \| \varphi \|_\mathcal{F})$
- $\| [\neg \varphi] \psi \|_\mathcal{F} \coloneqq \| \varphi \|_\mathcal{F} \to \| \psi \|_{\mathcal{F} \varphi}$
Everything is boolean!

Definition

Fix a knowledge structure $\mathcal{F} = (V, \theta, O_1, \cdots, O_n)$. We define a local boolean translation $\parallel \cdot \parallel_{\mathcal{F}}$:

- $\parallel p \parallel_{\mathcal{F}} := p$
- $\parallel \neg \varphi \parallel_{\mathcal{F}} := \neg \parallel \varphi \parallel_{\mathcal{F}}$
- $\parallel \varphi_1 \land \varphi_2 \parallel_{\mathcal{F}} := \parallel \varphi_1 \parallel_{\mathcal{F}} \land \parallel \varphi_2 \parallel_{\mathcal{F}}$
- $\parallel K_i \varphi \parallel_{\mathcal{F}} := \forall (V \setminus O_i)(\theta \to \parallel \varphi \parallel_{\mathcal{F}})$
- $\parallel [! \varphi] \psi \parallel_{\mathcal{F}} := \parallel \varphi \parallel_{\mathcal{F}} \to \parallel \psi \parallel_{\mathcal{F}^\varphi}$

Theorem

For all scenarios $(\mathcal{F}, s)$ and all formulas $\varphi$:

$\mathcal{F}, s \models \varphi \iff s \models \parallel \varphi \parallel_{\mathcal{F}}$
Example: Symbolic Muddy Children I

\[ \mathcal{F}_0 = \left( \begin{array}{c} V = \{p_1, p_2, p_3\}, \theta_0 = \top, \\ O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{array} \right) \]

“At least one of you is muddy.”
Example: Symbolic Muddy Children I

\[ \mathcal{F}_0 = \begin{pmatrix} \mathcal{V} = \{p_1, p_2, p_3\}, \theta_0 = \top, & O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} & O_3 = \{p_1, p_2\} \end{pmatrix} \]

“At least one of you is muddy.”

\[ \mathcal{F}_1 = \begin{pmatrix} \mathcal{V} = \{p_1, p_2, p_3\}, \theta_1 = (p_1 \lor p_2 \lor p_3), & O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} & O_3 = \{p_1, p_2\} \end{pmatrix} \]
Example: Symbolic Muddy Children I

\[ \mathcal{F}_0 = \begin{pmatrix} V = \{p_1, p_2, p_3\}, \theta_0 = \top, \quad O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{pmatrix} \]

“At least one of you is muddy.”

\[ \mathcal{F}_1 = \begin{pmatrix} V = \{p_1, p_2, p_3\}, \theta_1 = (p_1 \lor p_2 \lor p_3), \quad O_1 = \{p_2, p_3\} \\ O_2 = \{p_1, p_3\} \\ O_3 = \{p_1, p_2\} \end{pmatrix} \]

“Do you know if you are muddy?” … Nobody reacts.

This announcement is equivalent to:

\[ \| \bigwedge_{i \in I} (\neg (K_i p_i \lor K_i \neg p_i)) \|_{\mathcal{F}_1} = (p_2 \lor p_3) \land (p_1 \lor p_3) \land (p_1 \lor p_2) \]
Magic from 1986: Binary Decision Diagrams

(Read the classic Bryant 1986 for more details.)
BDD Magic I

How long do you need to compare these two formulas?

\[ p_3 \lor \neg(p_1 \rightarrow p_2) \equiv \neg(p_1 \land \neg p_2) \rightarrow p_3 \]
BDD Magic I

How long do you need to compare these two formulas?

\[ p_3 \lor \neg(p_1 \to p_2) \equiv \neg(p_1 \land \neg p_2) \to p_3 \]

Here are is their BDDs:
This was not an accident, BDDs are canonical.

**Theorem:**

$$\varphi \equiv \psi \iff \text{BDD}(\varphi) = \text{BDD}(\psi)$$

Equivalence checks are free and we have fast algorithms to compute $\text{BDD}(\neg \varphi), \text{BDD}(\varphi \land \psi), \text{BDD}(\varphi \rightarrow \psi)$ etc.
Results

Why translate DEL to boolean formulas?

Because computers are incredibly good at dealing with them!

<table>
<thead>
<tr>
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<th>DEMO-S5</th>
<th>SMCDEL</th>
</tr>
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So far, so good ... 

This was S5 PAL, what about:

1. Non-S5 relations for belief?

2. Action Models with factual change?
Motivating Example: Sally-Anne

This is Sally.

Sally has a basket.

Sally has a marble. She puts the marble into her basket.

Sally goes out for a walk.

Anne has a box.

Anne takes the marble out of the basket and puts it into the box.

Now Sally comes back. She wants to play with her marble.

Where will Sally look for her marble?
Motivating Example: Sally-Anne

To model this we need non-S5 action models with factual change!
Extension to non-S5

- $O_i \subseteq P$ always defines equivalence relations!
- How can we describe other relations?

- Fresh variables $V' := \{p' \mid p \in V\}$ and formulas $L(V \cup V')$
- Actually: use BDD of boolean formula describing the relation
Extension to non-S5

- $O_i \subseteq P$ always defines equivalence relations!
- How can we describe other relations?

- Copy $P$ to describe reachability (Gorogiannis and Ryan 2002)
- Fresh variables $V' := \{p' \mid p \in V\}$ and formulas $\mathcal{L}(V \cup V')$
- Actually: use BDD of boolean formula describing the relation
Extension to non-S5: From relations to BDDs

Relation $R$

Formula $\Phi(R)$.

BDD of $\Phi(R)$.
Extension to non-S5: Belief Structures

\[ \mathcal{F} = (V, \theta, \Omega_1, \cdots, \Omega_n) \text{ where} \]

- **V** \textit{Vocabulary} propositional variables
- **\theta** \textit{State Law} boolean formula over \( V \)
- **\Omega_i \in \mathcal{L}(V \cup V')** \textit{Observables} encoded relation for \( i \)

The equivalent Kripke model is given by:

\[ R_{i}xy : \iff (x \cup y') \models \Omega_i \]

New translation for modalities:

\[ \| \square_i \psi \|_{\mathcal{F}} := \forall V'(\theta' \rightarrow (\Omega_i \rightarrow (\| \varphi \|_{\mathcal{F}}')) \) \]
The return of the postcondition

1. Dynamic Epistemic Logic
2. Symbolic Model Checking
3. **Factual Change**
**Transformers**

**Definition**

A *transformer* for $V$ is a tuple $\mathcal{X} = (V^+, \theta^+, V_-, \theta_-, \Omega^+)$ where

- $V^+$ is a set of fresh atomic propositions s.t. $V \cap V^+ = \emptyset$
- $\theta^+$ is a possibly epistemic formula from $\mathcal{L}(V \cup V^+)$
- $V_- \subseteq V$ is the *modified subset* of the original vocabulary
- $\theta_- : V_- \rightarrow \mathcal{L}_B(V \cup V^+)$ encodes postconditions
- $\Omega^+_i \in \mathcal{L}_B(V^+ \cup V^{+\prime})$ describe observations for each $i$

To transform $\mathcal{F} = (V, \theta, \Omega_i)$, let $\mathcal{F} \times \mathcal{X} := (V^{\text{new}}, \theta^{\text{new}}, \Omega_i^{\text{new}})$ where

1. $V^{\text{new}} := V \cup V^+ \cup V^\circ$
2. $\theta^{\text{new}} := [V_-/V^\circ] (\theta \land \|\theta^+\|_{\mathcal{F}}) \land \bigwedge_{q \in V_-} (q \leftrightarrow [V_-/V^\circ] (\theta_-(q)))$
3. $\Omega_i^{\text{new}} := ([V_-/V^\circ] [(V_-)'/((V^\circ)') \Omega_i] \land \Omega_i^+$
Simple Example: Coin Flip hidden from a

\[
\begin{align*}
  V & = \{ p \}, \quad \theta = p, \quad \Omega_a = T, \quad \Omega_b = T \\
\times & \quad (V^+ = \{ q \}, \quad \theta^+ = T, \quad \Omega_a^+ = T, \quad \Omega_b^+ = q \leftrightarrow q') \\
\times & \quad (V_- = \{ p \}, \quad \theta_-(p) := q, \quad \Omega_a^- = T, \quad \Omega_b^- = q')
\end{align*}
\]
Monster Example: Symbolic Sally-Anne I

- Sally is in the room: $p$, Marble is in the box: $t$
- Question after the actions: Does Sally believe that the marble is in the box: $\Box_s t$?
Monster Example: Symbolic Sally-Anne I

- Sally is in the room: $p$, Marble is in the box: $t$
- Question after the actions: Does Sally believe that the marble is in the box: $\square_s t$?

Initial structure: $(((\{p, t\}, (p \land \neg t), \top, \top), p)$

$X_1$: Sally puts the marble in the basket:
$(((\emptyset, \top, \{t\}, \theta_-(t) = \top, \top, \top), \emptyset)$.

$X_2$: Sally leaves:
$(((\emptyset, \top, \{p\}, \theta_-(p) = \bot, \top, \top), \emptyset)$.

$X_3$: Anne puts the marble in the box, not observed by Sally:
$(((\{q\}, \top, \{t\}, \theta_-(t) = (\neg q \rightarrow t) \land (q \rightarrow \bot), \neg q', q \leftrightarrow q'), \{q\})$.

$X_4$: Sally comes back:
$(((\emptyset, \top, \{p\}, \theta_-(p) = \top, \top, \top), \emptyset)$. 
The slide we will skip

× \((\{p, t\}, (p \land \neg t), \top, \top), p)\)
= \(((\emptyset, \top, \{t\}, \theta_-(t) = \top, \top, \top), \emptyset)\)
= \(((\{p, t, t^\circ\}, (p^\circ \land \neg t^\circ) \land t \land \neg p, \top, \top), \{p, t\})\)
≡ \(\emptyset^\circ \times (\emptyset, \top, \{p\}, \emptyset, \emptyset)\)
= \(((\emptyset, \top, \emptyset, \emptyset), \emptyset, \emptyset, \emptyset, \emptyset)\)
= \(((\{p, t, q, t^\circ\}, t^\circ \land \neg p \land (t \leftrightarrow ((\neg q \rightarrow t^\circ) \land (q \rightarrow \bot))) \land \neg \neg q, q \leftrightarrow q'), \{q\})\)
≡ \(\emptyset^\circ \times (\emptyset, \top, \emptyset, \emptyset)\)
= \(((\emptyset, \top, \emptyset, \emptyset), \emptyset, \emptyset, \emptyset, \emptyset)\)

× \(((\emptyset, \top, \emptyset, \emptyset), \emptyset, \emptyset, \emptyset, \emptyset)\)
= \(((\emptyset, \top, \emptyset, \emptyset), \emptyset, \emptyset, \emptyset, \emptyset)\)
= \(((\{p, t, q, p^\circ\}, \neg p^\circ \land (t \leftrightarrow \neg q) \land p \land \neg \neg q', q \leftrightarrow q'), \{p, q\})\)
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= \(((\{p, t, q\}, (t \leftrightarrow \neg q) \land p \land \neg \neg q', q \leftrightarrow q'), \{p, q\})\)
Everything on the previous scary slide are boolean operations, so we blindly trust a computer to deal with it.
Monster Example: Symbolic Sally-Anne IIII

In the last scene Sally believes the marble is in the basket:

\[
\{p, q\} \models \Box_{\mathcal{S}} t
\]

\[\iff\]

\[
\{p, q\} \models \forall V'(\theta' \rightarrow (\Omega_{\mathcal{S}} \rightarrow t'))
\]

\[\iff\]

\[
\{p, q\} \models \forall \{p', t', q'\}((t' \leftrightarrow \neg q') \land p' \rightarrow (\neg q' \rightarrow t'))
\]

\[\iff\]

\[
\{p, q\} \models \top
\]

(François Schwarzentruber: *Hintikka’s world*)
Summary

- representation matters!
- *symbolic* model checking DEL gives a big speed-up
- *knowledge/belief structures* encode Kripke models
- *transformers* provide a modular approach for ontic and epistemic actions

Future Work

- Comparison with (Charrier and Schwarzentruber 2017)
- Find big examples and benchmark!
- Limit postconditions to $\top$ and $\bot$?

github.com/jrclogic/SMCDEL

malvin@w4eg.eu
Similar approach in (Charrier and Schwarzentruber 2017). *Mental programs* describe which change is allowed: $q \mapsto \top, p \mapsto q, \ldots$

- Observational BDDs and mental programs are dual in memory consumption:

<table>
<thead>
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<th>Relation</th>
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<th>Mental Program</th>
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<tbody>
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<td>$2 \cdot</td>
<td>V</td>
</tr>
<tr>
<td>Total</td>
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<td>$2 \cdot</td>
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